Periodic Section Modeling of Convective Continuous Powder Mixing Processes

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This study aims to develop a general model of the convective continuous mixing process. The main idea is that continuous mixing can be considered as a combination of powder flow and mixing processes. Although powder flow is characterized by the residence time distribution (RTD), powder mixing can be described by a batch mixing process simulated in one periodic section of the continuous mixer. By characterizing the two processes separately, we can calculate the number of sections required to achieve certain homogeneity. In this study, continuous mixing is simulated using the discrete element method, and segregating and non-segregating mixing cases are tested to investigate the applicability of the model. Results show satisfactory predictions by the model, which is able to characterize the continuous mixing performance of both mixing cases. On the basis of this study, we were also able to suggest a novel method in design and control of continuous powder mixing systems. © 2011 American Institute of Chemical Engineers *AIChE J*, 58: 69–78, 2012
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Introduction

Powder mixing is widely used in many industries in which reduced heterogeneity in the product mixture is essential for further manufacturing. In pharmaceutical industry, mixing has been operated traditionally in batch mode as it guarantees accurate feeding as well as provides reliable quality control. However, as the scaling up is difficult and frequent manipulations are necessary in the operation of batch process, in recent years studies have been focused on continuous mixing, which is considered as an appealing alternative suitable for high flux manufacturing.[1]

Many different types of equipment are available to perform continuous mixing. In fluidized bed mixers, mixing is achieved by the random movement of particles in "gravity free" conditions caused by the pressurized fluid moving upward through the particle medium. Although this process unit is typically used in batch mode, it has been applied in continuous mixing where continuous feeding and removing of powders are desired.[2] In rotary cylinder mixers,[3,4] particles move forward due to the fill level gradient through the cylinder. Mixing takes place around the cylinder surface due to the frictional stress between the surface and the particles,[5] which can be improved when lifters are involved in the process.[6] Convective continuous mixers are closer in design to rotary cylinders with lifters.[7] They consist of impellers rotating around a shaft, which cause particle displacement to the regions they cannot reach through free flowing movement. Although convective mixers can be operated in either batch or continuous mode, different blade configurations and rotary speeds can cause dense or fluidized particle movement, which makes this kind of mixer suitable in processing both free-flowing and cohesive particles.

One of the earliest studies on continuous mixing addressed the influence of different types of mixtures on process performance.[8] This study investigated the continuous mixing of cohesive and free-flowing particles, which shows that the performance of continuous mixer is determined at least in part by the flow properties of materials. The influence of the agitator design on powder flow has been also studied,[9] where

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the velocity profiles of the particle flow were characterized using positron emission particle tracking (PEPT). The influence of stirrer types on mixing heterogeneity was studied for different feed rates and blade speeds.[10] Investigation of the effect of different operating conditions on the resulting mixture quality was performed in previous studies,[7] indicating that the cohesive properties of material did not significantly affect the convective mixing performance. Although it is clear that the role of continuous powder mixer is to mix initially segregated fluxes (radial mixing) as well as to attenuate feeding fluctuations (axial mixing),[11] the theoretical basis for design and optimization of continuous mixers, and especially the radial mixing component, requires additional work. In many of the previous studies, the continuous mixers were considered as black boxes, that is, variables were changed and responses at the outlet were observed. Since this investigating method cannot provide information about the transitional behavior in the process of mixing, questions such as whether radial mixing is complete at the outlet or whether the mixing efficiency is optimized still cannot be answered, which hinders the optimization and control of this process.

To elucidate the mixing performance of the mixer, this study assumes convective continuous mixing as a combination of transverse mixing and axial movement. Moreover, it is assumed that the mixing process shares similar characteristics as a batch mixing process within any transverse periodic section of the mixer. The axial movement can be characterized by the residence time distribution of the mixer. Thus, mixing along the axis of a continuous mixer composed of a series of periodic sections can be estimated. Non-segregating and segregating particles are used to test this idea. The rest of the article is organized as follows: the method, the description of the DEM environment, and the simulation design are presented in *Materials and Methods*, followed by the introduction of the *periodic section modeling*. *Results* are discussed in the next section, which shows satisfactory prediction of both non-segregating and segregating continuous mixing processes. We conclude with the applicability of the modeling in "*Conclusion*."

Materials and Methods

Periodic section

As indicated in the introduction, batch mixing inside one periodic section of a convective continuous mixer is used in this article to illustrate the continuous mixing process. The main characteristic of the proposed periodic section is that each particle that exits through one side of the periodic section re-enters the section from the other side. This periodicity results in the main advantage of this method in studying the flow pattern of the continuous mixer: because the volume of the section is smaller than a whole mixer, the approach allows faster simulations with fewer particles, which limits the computational cost. The effects of operating conditions on flow and dispersion of continuous mixing process has been studied based on the utilization of periodic section.[12] Earlier, this approach was used to increase the resolution of liquid mixing simulations in static mixers.[13,14] On the basis of these studies, the batch mixing performance of periodic section is discussed here.

In this work, the convective continuous mixing process is regarded as the combination of the batch mixing in one of the periodic sections in the continuous mixer, and its axial movement along the whole mixer. On one hand, the axial movement is consisted of the axial velocity v_x and dispersion coefficient E_x, resulting in a residence time distribution at the outlet; on the other hand, we consider the periodic section as a batch due to the definition of batch mixing, in which no net exit or entrance of particles takes place once mixing starts. Since particles are in the same space through the process, mixing in the periodic section is only a function of time, the same as in batch. Meanwhile, steady state can be reached in a continuous mixer composed of a series of periodic sections. In this case, the distance the powder travels, instead of the time the particles stays inside the batch mixer, can be regarded as the measure of mixing process.

Table 1. Physical and Numerical Parameters Used in the DEM Model

Parameter	Value		
	Particle 1	Particle 2	Particle 3
Particle diameter	3×10^{-3} m	2×10^{-3} m	4×10^{-3} m
Particle density	1×10^{3} kg/m^3	2×10^{3} kg/m^3	0.5×10^{3} kg/m^3
Poisson's ratio		0.25	
Shear modulus		3×10^{8} Pa	
Coefficient of restitution		0.5	
Coefficient of static friction		0.5	
Coefficient of rolling friction		0.01	
Simulating time step		10^{-6} s	
Recorded time step		0.01 s	
Sampling time interval		0.1 s	

Computational environment

It is difficult to experimentally design a system that allows investigation of the batch-like mixing in a periodic section. Instead, the discrete element method (DEM) is used to simulate the process. DEM refers to a family of numerical methods for computing the motion of large number of particles like molecules or grains of sand, by solving Newton's equation of motion for each particle. In this work, the software EDEM® from DEM Solutions is the computational environment used to develop the desired geometry and model of particle motion. In this work, particles are modeled as monodisperse spheres, whereas the contact forces are calculated using the Hertz-Mindlin no slip contact model.[15] The physical and numerical parameters used in this simulation are summarized in Table 1. The values are chosen so that complete mixing can be achieved in the time interval of simulation.

Materials, geometry, and simulation studies

To investigate different mixing behaviors, three kinds of particles with different diameters and densities were used in the simulation (Table 1) to investigate how segregating and non-segregating particles influence the relationship between mixing in the periodic section and in the corresponding continuous mixer. In the present study, we investigate two case

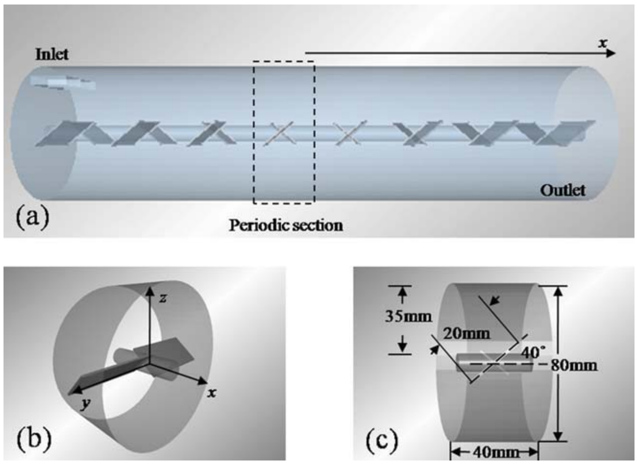

Figure 1. Simulated geometry of the mixer composed of eight periodic sections.

(a) Side view of the mixer, blades rotate counter-clockwise and push particles forward along x axis, from the inlet to the outlet, (b) isometric view of the periodic section, and (c) side view of the periodic section with displayed dimensions. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

studies: a first mixture consisting of only particles Type 1, having different colors, and a second mixture of particles Types 2 and 3. The first case is representative of a non-segregating mixture, whereas the second case will result in segregation, where the smaller, higher density Type 2 particles will sink downward through the gaps between the larger, lighter Type 3 particles.

To compare the similarity between batch-like mixing in the periodic section and the corresponding continuous mixing in a full blender, simulations of both processes are performed. In the current study, a convective continuous mixer composed of eight sections with the same geometry was considered. The mixing zone consists of a horizontal cylinder, in which two opposing blades are located on the inner shaft at the middle of each section (Figure 1). The detailed geometry of these sections is shown in Figures 1b, c. Blades are oriented 40° at the cylinder axis to propel particles forward, as well as to mix particles in the transverse directions. Particles are fed side-by-side continuously from the top of the first section, and leave the mixer at the open end on the right side of the last section (Figure 1a). The left side of the mixer is closed to avoid material leakage. In steady state, feed rate at the inlet should equals to the flow rate everywhere inside the mixer. As no weir is used at the outlet of the mixer, no backward flux should be observed in the last several sections. This indicates similar flow patterns (all forward) in the continuous mixing case, and in the batch mixing case of the corresponding periodic section. Meanwhile, the absence of weir could lead to much lower fill level in the last two or three sections than the rest. As mixing efficiency depends significantly on fill level,[12,16] the mixing efficiency in the last several sections should be different from the rest. This will be considered in the development of the periodic section modeling in "Methods."

The batch mixing in the corresponding periodic section also needs to be characterized. Notice that the periodic section shares the same geometry with each single section of the full mixer (Figures 1b, c). Instead of feeding continuously at the top of the section, particles are initially loaded

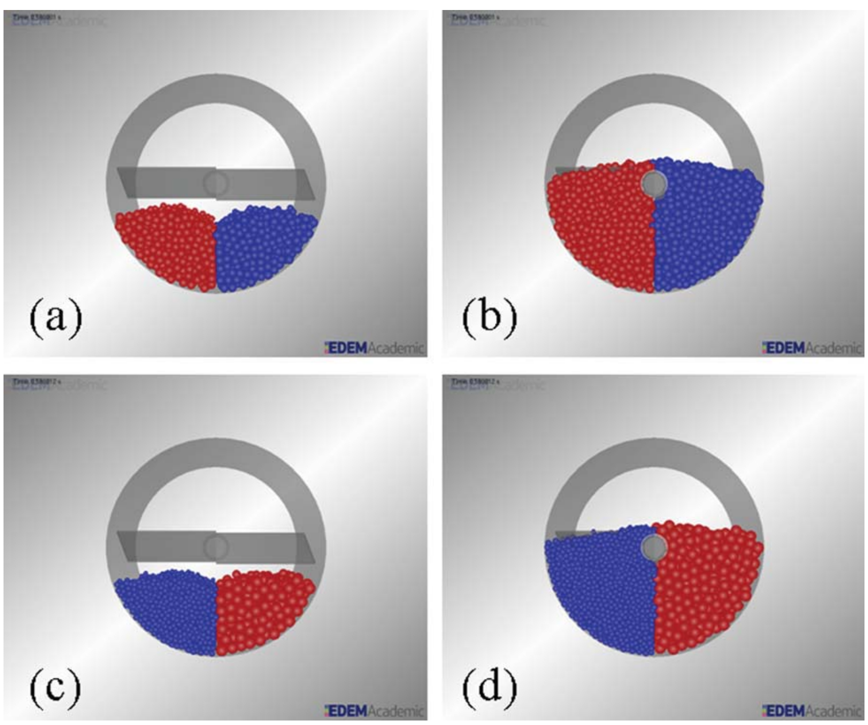

Figure 2. Initial loading of batch mixing in periodic section.

(a) 25% Fill, non-segregating mixture, (b) 50% fill, non-segregating mixture, (c) 25% fill, segregating mixture, and (d) 50% fill, segregating mixture. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Table 2. The Number of Initial Loading Particles for the Periodic Section

	Particle Number (N)			
	Non-Segregating Particles		Segregating Particles	
Fill Level	Type 1 (blue)	Type 1 (red)	Type 2 (blue)	Type 3 (red)
25%	1120	1120	3840	480
50%	2240	2240	7680	960

side-by-side through the blade shaft (Figure 2). This insures that the main compositional gradients at the beginning of the mixing process are in the same transverse direction as the corresponding continuous process where different materials are fed in a similar side-by-side style. The loaded fill level and component ratio in the periodic section are set to be the same as the steady state continuous mixing. Although batch mixing takes place in the periodic section, a net forward flux due to the two forward blades is generated in the mixing space. This leads to flow pattern and mixing condition in the periodic section similar to those in the corresponding continuous case. However, for the periodic section, the time-dependent batch mixing process needs to be converted to a location-dependent continuous process to characterize the mixing of the whole mixer.

Simulations were done at the blade speed of 180 RPM and the volumetric ratio of components 1:1. Two fill levels (25% and 50%), two sampling sizes (0.15 cm^3 and 0.75 cm^3) and two mixing pairs (segregating and non-segregating mixtures) were investigated in the study. As a result, four pairs of mixing processes were investigated in two sampling sizes, and were used to model the mixing process for both mixtures. The processes are performed and measured as follow.

Methods

Fill Level and Flow Rate. To be able to match the fill level and the feed rate of the periodic section and the continuous mixer the following definition is used for the fill level:

$$\text{fill-level} = \frac{\pi d_p^3 \Delta n/6}{0.64 \cdot \pi (D_{\text{mixer}}^2 - D_{\text{shaft}}^2)/4} \tag{1}$$

where Δn indicates the number of particles per length in the axial direction, d_p the particle diameter, D_{mixer} and D_{shaft} are the diameter of the cylinder and the inner shaft, respectively. The coefficient 0.64 is involved as the random packing solid fraction for spheres. Note that the unit of fill level in Eq. 1 is percentage (%). Equation 1 defines the (volumetric) fill level as the ratio of the volume occupied by the random packed particles to the volume of the cylinder (excluding the volume of the inner shaft). When the size of the two types of particles are different (mixing Case 2), the calculation is made by converting them to only one type of particle by keep constant volumetric occupation. On the basis of Eq. 1, the initial loading number of each type particles $N = \Delta n l_0 \overline{C}$ in the periodic section can be derived for both fill levels (Table 2). Here, $l_0 = 40$ mm is the length of one periodic section, and $\overline{C} = 50\%$ is the volumetric fraction of different types of particles.

When batch mixing is complete, the average axial velocity v_x is calculated, which leads to the flow rate F:

$$F = \Delta n v_x \tag{2}$$

where F is measured as number of particles per time. It should be noticed that F also represents the feed rate in the corresponding continuous mixing that leads to the same fill level. Thus, Eq. 2 is used to determine the required feed rates in the corresponding simulations of continuous mixing (Table 3).

Measurement and Model of Variance and RSD. To determine the time-dependent change in the degree of heterogeneity of the mixture, samples are retrieved inside the mixing space. Cubes with the side length 10 mm were divided in both the continuous mixer and the periodic section. In the cases of periodic section (batch mixing), all the samples in each interval of 0.1 s of the simulation process were considered as the samples of the same time point, which guarantees that enough samples (80 or more) were retrieved in each time point. In the cases of continuous mixing, the samples located in the same plane perpendicular to the x axis were regarded as the samples in the same location. The time interval recorded is also 0.1 s. To investigate the influence of the sampling size, samples were selected based on the number of particles inside each cube. Thus, for the non-segregating case (particle Type 1), samples with 7 or 35 particles were selected, respectively, corresponding to the number of particles that can be randomly packed within the sampling sizes of 0.15 cm^3 or 0.75 cm^3. For the segregating case (particle Types 2 and 3), since particle sizes are different, samples with particles volumetrically equivalent to 23 and 118 particles Type 2 were selected, to represent different sampling sizes. It should be noticed that both sampling sizes are relatively smaller than in practice, so that their influence on the value of RSD measurement can be clearly distinguished.

After selecting the samples, the variance (σ^2) and the relative standard deviation (RSD) of the volumetric fraction in samples of each time point or location can be calculated and used as a measure of the mixture heterogeneity:

$$RSD = \frac{\sigma}{\overline{C}} = \frac{\text{Standard deviation}}{\text{Average concentration}}; \sigma^2 = \frac{\sum_1^N (C_i - \bar{C})^2}{N-1} \tag{3}$$

where σ^2 is the variance, $\overline{C}$ is the average volumetric fraction of N (80 or more) samples, and C_i is the fraction of each sample.

To characterize the batch-like mixing process with fewer parameters, the variance profile $\sigma_b^2(t)$ was modeled by the exponential decrease relationship shown below:

$$\sigma_b^2(t) = \sigma_{SS}^2 + (\sigma_0^2 - \sigma_{SS}^2)\exp(-k_b t) \tag{4}$$

Table 3. Feeding Conditions for the Corresponding Continuous Mixer

	Feed Rate F (particles/s)					
	Non-Segregating Particles			Segregating Particles		
Fill Level	v_x(m/s)	Type 1 (blue)	Type 1 (red)	v_x(m/s)	Type 2 (blue)	Type 3 (red)
25%	0.121	3390	3390	0.119	11,424	1428
50%	0.104	5820	5820	0.104	19,968	2496

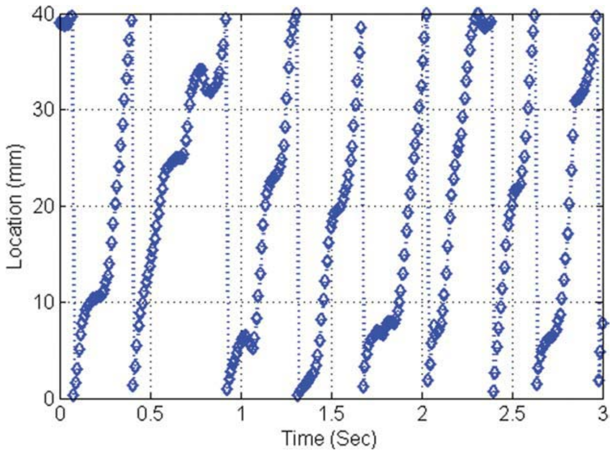

Figure 3. Trajectory of one particle inside the periodic section.

The time interval between two transverses through one end of the periodic section (adjacent vertical dash lines) was used as one sample in calculating the RTD. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

where $\sigma_b^2(t)$, σ_{SS}^2, and σ_0^2 are the variance in batch mixing of periodic section, the steady state variance of the investigated sampling size, and the initial variance when particles are totally segregated, respectively; and k_b is the variance decay rate of the batch mixing process. Note that similar relationships has been used in previous studies to model V blenders, bin blenders, and other batch systems.[17,18] As $\overline{C} = 0.5$, σ_0^2 can be considered as equal to 0.25 using the following equation:

$$\sigma_0^2 = \bar{C}(1 - \bar{C}) \tag{5}$$

which leads to the initial RSD value of $RSD_0 = 1$ by applying Eq. 3.

Measurement and Model of RTD. To characterize particle movement of both the continuous mixer and the periodic section, the residence time distribution (RTD) is used in this work. Instead of using the pulse test method by Danckwerts[19] in the simulation of continuous mixing, RTD was measured by considering the trajectory of particles inside the flow of the mixer, or the periodic section. Figure 3 shows the trajectory of one particle in the direction of axial movement in the periodic section. Since the blades in the periodic section are all forward, unilateral movement can be observed for the studied particle. When it reaches the right side of the section (40 mm), it reappears at the other side (0 mm). Therefore, the time interval between two transverses at one end of the space can be considered as one sample of the RTD of a single periodic section. As a result, the RTD can be derived using the following equation:

$$E(t_i) = \frac{n_i}{\sum\limits_{i=1} n_i \Delta t} \tag{6}$$

where $E(t_i)$ is the RTD at the ith time point t_i, Δt the sampling time interval, and n_i the number of samples in which the residence time of each sample lies in (t_i, $t_i + \Delta t$). The RTD within sections of a whole continuous mixer is measured in a similar way. However, in this case, one particle reaches the right side of the sections will leave the system, instead of reappearing at the left side. To eliminate noise from the measurements of RTD, around 6000 samples of particle trajectory were retrieved for each RTD measurement. If the same flow rate and fill level are shared by sections in a continuous mixer and the corresponding periodic section, the RTD within the continuous mixer can also be derived through the convolution of the RTD in one periodic section for several times, which allows more efficient calculation.

To clarify the physical meaning of the statistical RTD obtained above, the Taylor dispersion model is introduced as follows:

$$E(\xi, \theta) = \frac{Pe^{1/2}}{(4\pi\theta)^{1/2}} \exp\left(\frac{-Pe(\xi - \theta)^2}{4\theta}\right) \tag{7}$$

where $\theta = t/\tau$ and $\xi = x/l$ represent the dimensionless time and dimensionless location in the continuous mixing process, respectively; τ is the mean residence time and l is the length of the studied flowing system; $Pe = v_x l / E_x$ is the Peclet number; v_x and E_x are the axial velocity and dispersion coefficient of the system. On the basis of the Taylor dispersion model, the statistical RTD data is fitted to get a smoothed curve, which will be applied into the periodic section modeling proposed below.

Periodic Section Modeling

As indicated in the previous section, the decrease of mixture heterogeneity is time-dependent in the batch mixing process of the periodic section whereas it is location-dependent in the continuous mixing process. Meanwhile, we assume approximately a constant mixing efficiency through the whole continuous mixer where the fill level is almost the same, except for the last several sections. This indicates that in these first several constant-fill sections, the continuous mixing process of one cluster of particles depends only on the time the cluster spends inside these sections. In other words, a continuous mixer with a series of constant-fill sections can be considered as a batch mixer, which has the same fill and mixing efficiency, and is operated for a time period equivalent to the RTD in the continuous mixer. Thus, the present model should be designed to link the mixing efficiency of these constant-fill sections, the RTD the clusters of particles stay within these sections, and the heterogeneity decay curve along the continuous mixer. Since the batch mixing in one periodic section should share the fill level and flow pattern with continuous mixing within these constant-fill sections, we assume they share approximate the same mixing efficiency. Based on this assumption, as well as the definitions of RTD and variance in the previous section, the following empirical relationship is proposed:

$$\sigma_c^2(x) = \int_0^\infty \sigma_b^2(t) E(t, x) dt \tag{8}$$

where $\sigma_c^2(x)$ is the variance at different locations x along the axis of a continuous mixer; $\sigma_b^2(t)$ is the variance decay of the batch mixing in the periodic section; and $E(t,x)$ is the RTD

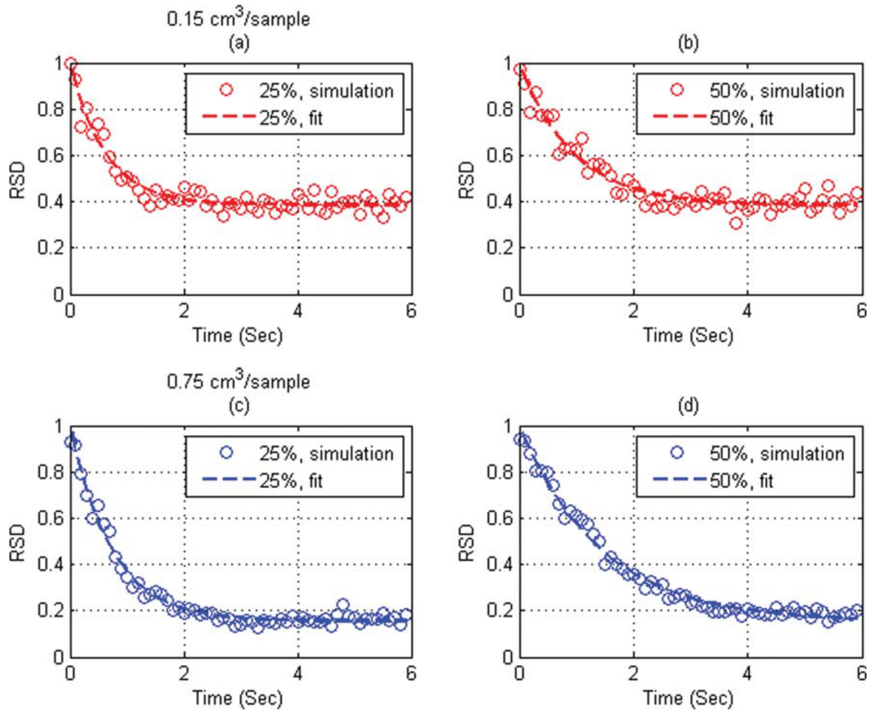

Figure 4. RSD-*t* curves of batch mixing in periodic section of non-segregating particles.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

in the continuous mixer at location x. Notice that the sampling sizes of the measurement of $\sigma_c^2(x)$ and $\sigma_b^2(t)$ should be the same.

Equation 8 is proposed based on the assumption that the variance at each transverse plane in the continuous mixer is the average of the variance of the clusters of particles at the same plane, which may have spent different residence time inside the mixer. Since in most cases, the RTD in the convective continuous mixer is unimodal (single peak), the average variance of these clusters of particles should be close to the variance of the cluster with the most particles, or the cluster that spends the mean residence time inside the mixer. On the basis of this observation, Eqs. 4 and 8 can be re-written as follows:

$$\sigma_c^2(x) = \sigma_{SS}^2 + (\sigma_0^2 - \sigma_{SS}^2)\exp(-k_c x) \quad (9)$$

where σ_{SS}^2, σ_0^2 represent the same parameters as in Eq. 4, and k_c is regarded as the variance decay rate along the axis of a continuous mixer, which is calculated as:

$$k_c = k_b / v_x \quad (10)$$

Equations 8–10 show the potential relationship between the batch mixing of a periodic section and the corresponding continuous mixing, which shares the same fill level with the periodic section. Considering the difficulty on realizing the periodic section experimentally, Eq. 10 provides an alternative to estimate the value of k_b in practice. Since the variance decay and RTD data can be measured in the continuous mixing process, we can achieve the values of k_c and v_x conveniently, and then the value of k_b can be easily calculated. On the other hand, since RSD is usually used as mixing index in practice, we divide Eqs. 4, 8, and 9 by square of the average concentration ($\overline{C}^2$), and the equations are converted to:

$$RSD_b^2(t) = RSD_{SS}^2 + (RSD_0^2 - RSD_{SS}^2)\exp(-k_b t) \quad (11)$$

$$RSD_c^2(x) = \int_0^{\infty} RSD_b^2(t)E(t,x)dt \quad (12)$$

$$RSD_c^2(x) = RSD_{SS}^2 + (RSD_0^2 - RSD_{SS}^2)\exp(-k_c x) \quad (13)$$

The simulation designs described in the previous section are used in the next section to validate Eqs. 11–13.

Results

Non-segregating particle mixture case

The non-segregating batch mixing results of the periodic section are shown in Figure 4. The influence of the fill level and the sampling size can be charcterized. The upper graphs show the RSD decline at the smaller sampling size (7 particles), which indicates larger RSD_{ss} ($\sim$0.4). When decreasing fill level from 50% to 25%, the kinetics of mixing improves, that is, the RSD decays quicker. However, RSD_{ss} is independent of the fill level. This is expected since the final mixing state in non-segregating mixing case represents random packing of the mixture, in which RSD_{ss} is only related to the sampling size. In this case, RSD_{ss} can be calculated using the binomial theorem.[17] Similar results were observed at the larger sampling size, in which a smaller RSD_{ss} ($\sim$0.16) was obtained.

Table 4. Batch Mixing Parameters of Non-Segregating Particles

Sample Size (cm^3/sample)	25% Fill		50% Fill	
	RSD_{ss}	k_b (s^{-1})	RSD_{ss}	k_b (s^{-1})
0.15	0.39	1.62	0.38	1.25
0.75	0.16	1.95	0.15	1.13

To identify the model parameters for the batch-like processes, we use the least square method for the error between the simulation of RSD decay and the model (Eq. 11) for one component in the section. Since the initial state can be calculated using Eq. 5 ($RSD_0 = 1$), the parameters in present fitting process are only RSD_{SS} and k_b. The curves in Figure 4 correspond to the best fits and were obtained with parameters listed in Table 4. As expected, almost the same RSD_{ss} are achieved for both fills in non-segregating mixing whereas the larger value of k_b in 25% fill indicates faster mixing. In fact, based on Eq. 11, if $(RSD_b^2(t) - RSD_{SS}^2)/(RSD_0^2 - RSD_{SS}^2)$ is plotted logrithmatically vs. time, the batch mixing rate k_b can be defined as the negative slope of the linear relationship. Therefore, only two adjustable parameters are required to capture all process configurations and the fitted, smoothed decay curves can be used in predicting the mixing of the corresponding continuous processes.

The fill level distributions of the non-segregating mixing cases vs. the section number are also plotted (Figure 5a). Results indicate similar distributions for both fills. Because of the effect of no weir at the end of the mixer, fill level drops quickly at the right of the sixth section (the vertical dot line). Before that transverse plane, the fill levels are approximately constant. As indicated in the modeling, application of Eqs. 12 and 13 is valid only within constant-fill sections of the mixer. Therefore, the RTD only within the first six sections are statistically calculated, which are used in the corresponding continuous mixing prediction. The RTD data at 2, 4, 6 constant-fill sections are plotted in Figure 6. As the calculations from continuous mixing and from convolution of single periodic section mixing lead to the same RTD data, only the RTD from the continuous mixing are shown. Linear increase of the mean residence time, as well as the accumulated dispersion, can be observed with the increase of section number. On the other hand, the decrease of fill level leads to smaller mean residence time and boarder RTD profile. This indicates faster axial mixing and movement at low fill, which is often observed in both batch and continuous mixers.[7] As more space is available at low fill, a single particle experiences more turns-over per blade pass, which facilitates both axial flow and dispersion. The curves of RTD in Figure 6 represent the best fit by applying the least square method, and show good accuracy on describing the data.

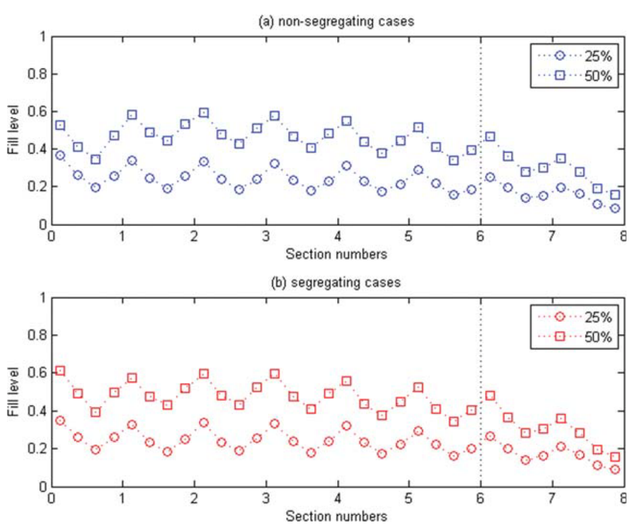

Figure 5. The fill level distribution along the mixing axis.

(a) Non-segregating cases and (b) segregating cases. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

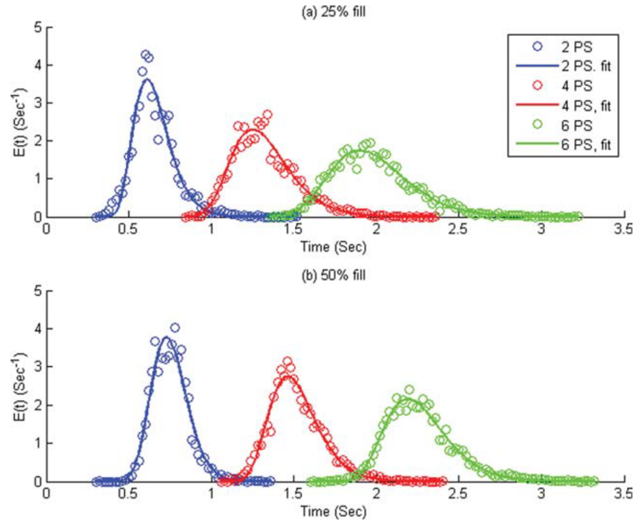

Figure 6. RTD data and curve-fitting of 2, 4, and 6 constant-fill sections for nonsegregating particles.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Figure 7 shows the simulated and calculated results of RSD decay in the case of continuous mixing, for the first six constant-fill sections. By using Eq. 12, the model fitted curves of both the batch mixing (RSD-t curve) in the periodic section and the RTD were applied in predicting the RSD decay curve within the continuous mixer. It can be seen that the calculated results could capture very well the simulated results for the constant-fill sections (Eq. 13 leads to almost the same prediction). To compare the RSD decay rate of different fills, the RSD values of 0.5 (for the sampling size of 0.15 cm^3) and 0.25 (for 0.75 cm^3) were considered as the check points. It can be observed at 25% fill, fewer periodic sections are needed to achieve the same check point, indicating better continuous mixing performance. On the basis of the obtained RSD-t and RTD information, although at 25% fill mixing is faster, the particles move also faster than that of 50% fill and exit the mixer earlier. Therefore, the influence of batch mixing rate k_b to the RSD decay rate $k_c = k_b/v_x$ is more significant than that of the particle movement v_x in the current mixing case. On the other hand, feed rate at 50% fill is significantly larger than that at 25% fill (Table 3), which is preferred in high flux manufacture process.

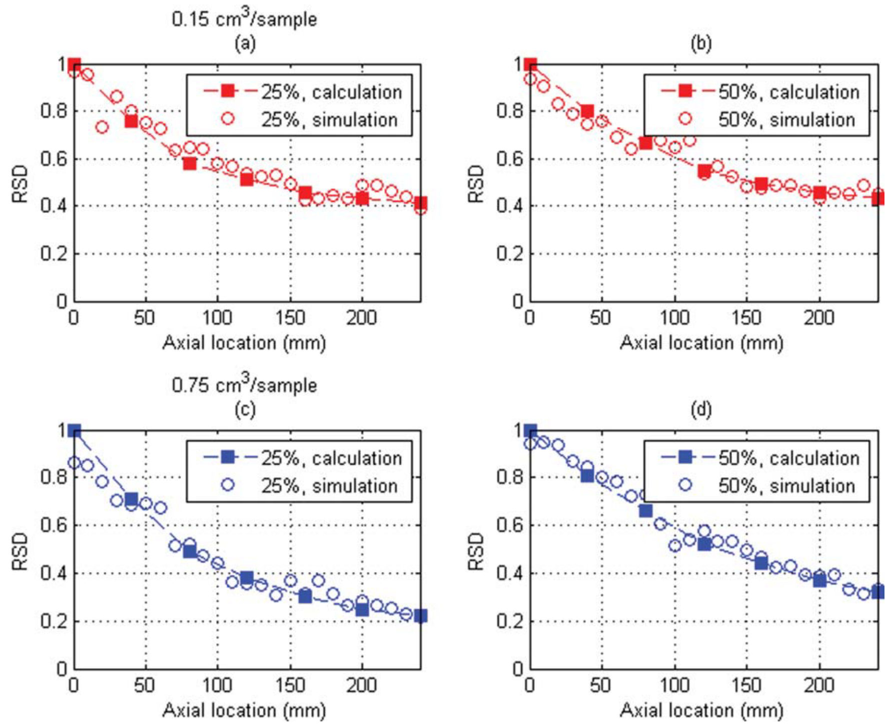

Figure 7. Simulation and calculation of RSD-l curves in continuous mixing processes, non-segregating particles.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Segregating particle mixture case

Figure 8 shows the batch mixing results for the segregating mixture. As illustrated in the previous section for the non-segretating case, decreasing the fill level allows larger batch mixing rate. However, the important aspect here is the fact that RSD_{SS} are fill level dependent, and in both fills should be higher than the case of random packing. This means that the size and density ratio between the particles

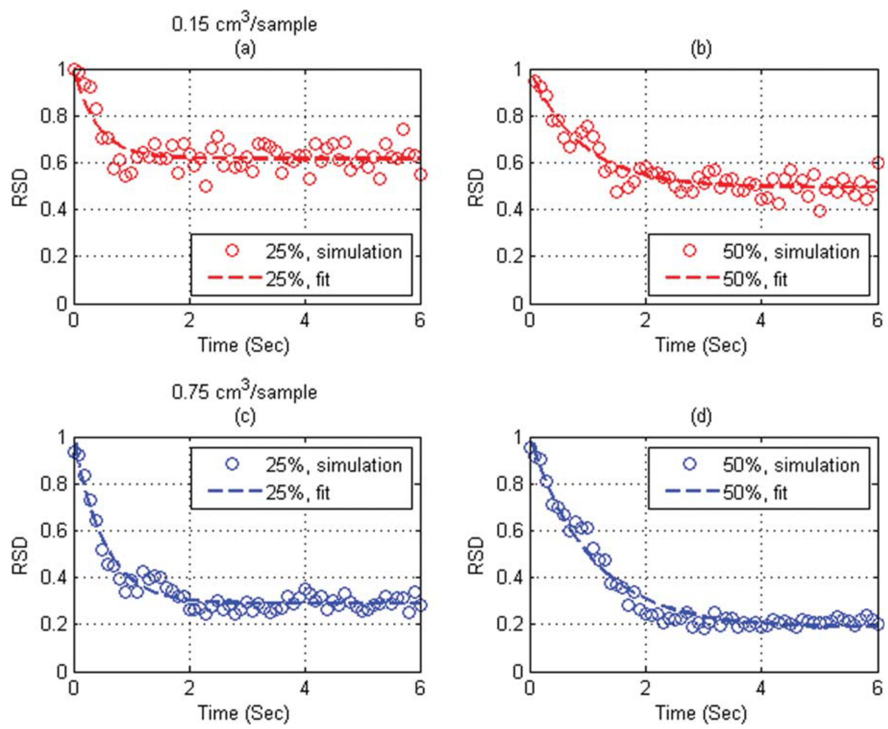

Figure 8. RSD-*t* curves of batch mixing in periodic section filled by segregating particles.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Table 5. Batch Mixing Parameters of Segregating Powders

Sample Size (cm^3/sample)	25% Fill		50% Fill	
	RSD_{ss}	k_b (s^{-1})	RSD_{ss}	k_b (s^{-1})
0.15	0.62	2.85	0.49	1.39
0.75	0.29	2.71	0.19	1.48

induces segregation that the mixer cannot completely eliminate. Because of the model fitted parameters listed in Table 5, RSD_{SS} are larger at 25% fills in both sampling sizes, indicating more segregation. This can be explained by the strength of blade disturbance on the segregation. As the fraction of prepelled particles is smaller at low fill, the influence blade disturbance to the formation of segregation is smaller, leading to a more segregated mixture.

For the segregating cases, the fill level distributions within the continuous mixer processes are almost the same as the non-segregating cases (Figure 5b). As a result, the first six sections in the continuous mixer approximately share constant fill level with the periodic section. The RTD calculated at the end of the 2, 4, 6 sections are plotted in Figure 9, which shown high similarity to these of the non-segregating cases. This similarity indicates small difference between the bulk flows of materials with different properties, in our current study.

The simulated and calculated results of RSD decay along continuous mixing axis can be seen in Figure 10. Although fluctuations can be observed on the simulated data, which are caused by random formation of segregation along the direction of particle movement, the calcuated curves predict the simulation trends very well. As mentioned in the batch mixing case, RSD_{SS} is determined by the strength of blade distubance, which is lower at 50% fill. Meanwhile, due to the coupling effects of both mixing and axial motion, RSD of continuous mixing decays quicker in the case of 25% fill, whereas better mixing is achived in the case of 50% fill. Since 50% fill also leads to higher feed rate and thus larger production rate (Table 3), it should be selected for continously mixing of this segregating case, although more sections are needed to construct the desired mixer.

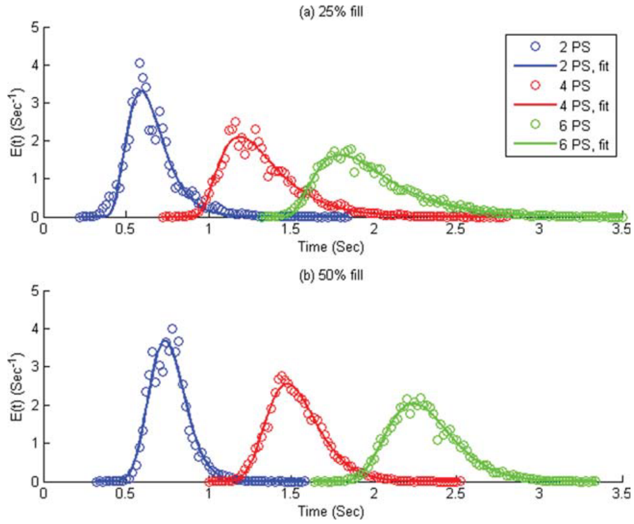

Figure 9. RTD of batch mixing in 2, 4, and 6 periodic sections for segregating particles.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

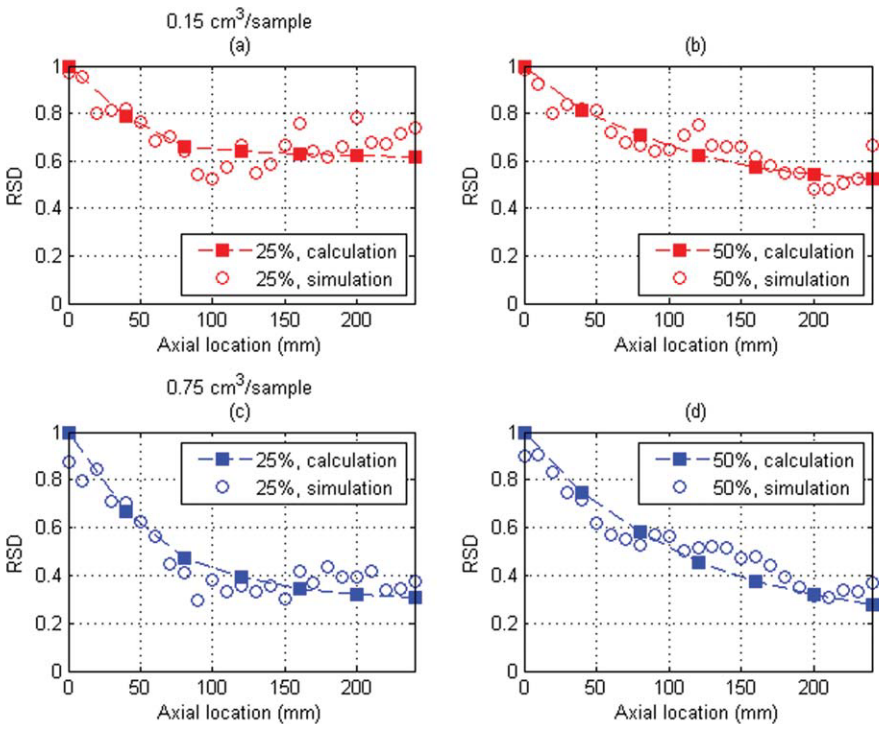

Figure 10. Simulation and calculation of RSD-l curves in continuous mixing processes, segregating particles.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Conclusions

In this study, we have proposed a model based on the idea that the convective continuous mixing results from the combination of a batch-like mixing and an axial flow of the particles. On the basis of the empirical relationship between the two processes, the cases of non-segregating and segregating mixing are investigated. The results show that the proposed methodology can describe both cases capturing the main characteristics of the continuous mixing process, including the batch mixing rate (k_b), the axial velocity particles move forward (v_x), and the quality of the steady state mixture (RSD_{SS}). Results show that the necessary number of periodic sections (indicated by k_c, larger k_c leads to less necessary number of sections) in the continuous mixing process is not only determined by the rate of local radial mixing, but also by the velocity how fast the materials are delivered inside the mixer. Therefore, for the purpose of clearly understanding effects of different variables on mixer performance, a separate consideration of mixing and moving is suggested for further study of continuous powder mixing processes.

In the application of the model, direct prediction of the variance (or RSD) decay requires constant-fill sections. If there is a fill level distribution (for instance, no weir at the end), mixing efficiency changes through the mixer, and prediction of the whole mixer through the batch mixing of periodic section at a single fill level is not applicable any more. That is the reason direct calculation using Eqs. 11 and 12 requires the exclusion of the sections with a different fill level. To improve this inconvenience, an important idea is that the variance decay can be calculated in separate subsections with different fills or mixing efficiencies. The decreased variance calculated at the end of one subsection should be used as the initial condition in the calculation of the next subsection. In other word, we can define a multiplicative process for the variance decay, where in each subsection, variance goes down by a factor $F_i = \exp(-k_{c,i}\Delta x_i)$. Here, $k_{c,i}$ represents the uniform variance decay rate along axis x within subsection i, and Δx_i denotes the length of the subsection. This requires involvement of mixing rates of the subsections at different fills, and the RTD calculated separately for different subsections. This issue is worthy of investigation and under consideration of future publication.

It should be emphasized that the periodic section modeling work indicates the similarity between the continuous powder mixing and an equivalent batch mixing process. Therefore, the knowledge developed in the field of batch mixing can be applied for the design and optimization of continuous mixers. That means that designs and operations that improve the performance of batch mixer can be conveniently used in the design of a continuous mixer, provided that the axial motion of the materials is slow enough for the equivalent batch mixing to be complete. Also, due to the difficulties of experimentally performing batch mixing of the periodic section, knowledge of the similarity between the mixing rate of one periodic section and batch mixers with similar geometries can facilitate the practical estimation of the mixing efficiency of a continuous process, and thus is worth considering.

Acknowledgments

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